ICCPP-STATISTICS

- Goodness of Fit Test

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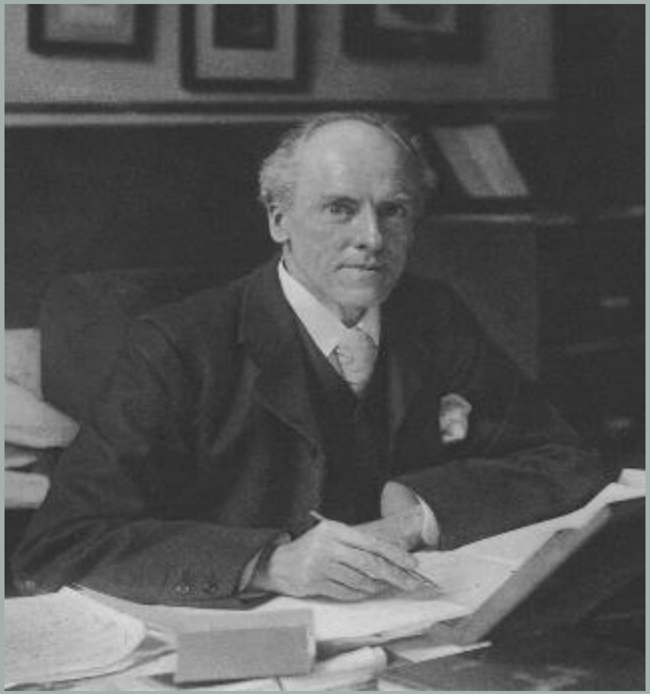

Karl Pearson (1857-1936)

Goodness of Fit Test

Definition

- The goodness of fit test is used to test if sample data fits a distribution from a certain population.

Goodness of Fit Test

- Goodness-of-fit tests are statistical tests to determine whether a set of actual observed values match those predicted by the model.

Most Common Goodness of Fit Tests

The goodness of fit test categorization can be done based on the distribution of the predictand variable of the dataset.

- The chi-square
- Kolmogorov-Smirnov
- Anderson-Darling

Chi-Square Goodness of Fit Test

- Chi-square goodness of fit test is conducted when the predictand variable in the dataset is categorical.
- It is applied to determine whether sample data are consistent with a hypothesized distribution.

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Chi-Square Goodness of Fit Test

Chi-Square test can be applied when the distribution has the following characteristics

- The sampling method is random.
- Predictand variables are categorical.
- The expected value of the number of sample observations at each level of the variable is at least 5. It requires a sufficient sample size for the chi-square approximation to be valid.

Merits of the Chi-square Test

- A distribution-free test. It can be used in any type of population distribution.
- It is widely applicable not only in social sciences but in business research as well.
- It can be easy to calculate and to conclude.

Merits of the Chi-square Test

- The Chi-Square test provides an additive property. This allows the researcher to add the result of independence to related samples.
- This test is based on the observed frequency and not on parameters like mean, and standard deviation.

Chi-square test for a goodness-of-fit test

$$\chi_c^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

Where:

- O_i = an observed count for bin $_i$,
- E_i = an expected count for bin $_i$, asserted by the null hypothesis.

Chi-square test for a goodness-of-fit test

- The expected frequency is calculated by:

$$E_i = \left(F(Y_u) - F(Y_l) \right) N$$

Where:

F = the cumulative distribution function for the probability distribution being tested.

Y_u = the upper limit for class $_i$,

Y_l = the lower limit for class $_i$, and

N = the sample size.

Applications

- The Chi-square is applied to establish or refute that a relationship exists between actual observed values and predicted values.
- It is used very commonly in Clinical research, Social sciences, and Business research.

Kolmogorov-Smirnov Goodness of Fit Test

- Andrey Kolmogorov and Vladimir Smirnov, two probabilists developed this test to see how well a hypothesized distribution function $F(_x)$ fits an empirical distribution function $Fn(_x)$.

Kolmogorov-Smirnov Goodness of Fit Test

- The Kolmogorov-Smirnov Goodness of Fit Test compares the dataset under consideration with a known distribution and lets us know if they have the same distribution.
- It’s also used to check the assumption of normality in Analysis of Variance.
- (KS-test) test can be applied when the Predicted variable is continuous.

Merits of (K-S-test)

- It does not make any assumptions about the distribution of data.
- It is widely applicable not only in social sciences but in business research as well.
- There are no restrictions on sample size; Small samples are acceptable.

K-S test for a goodness-of-fit test

- This test is used to decide if a sample comes from a hypothesized continuous distribution.
- It is based on the empirical cumulative distribution function (ECDF).

K-S test for a goodness-of-fit test

- Assume that we have a random sample x1, … , xn from some continuous distribution with CDF F(x).
- The empirical CDF is denoted by

$$F_n(x) = \frac{1}{n} \cdot \left[\text{Number of observations} \leq x\right]$$

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K-S test for a goodness-of-fit test

- The Kolmogorov-Smirnov statistic (D) is based on the largest vertical difference between $F(_x)$ and $Fn(_x)$.
- It is defined as a

$$D_n = \sup_x |F_n(x) - F(x)|$$

K-S test for a goodness-of-fit test

- H_0: The data follow the specified distribution.
- H_A: The data do not follow the specified distribution.
- The hypothesis regarding the distributional form is rejected at the chosen significance level (alpha) if the test statistic D is greater than the critical value obtained from a table.

Anderson-Darling Goodness of Fit Test

- The Anderson-Darling is tested to compare the fit of an observed cumulative distribution function to an expected cumulative distribution function.
- Like the K-S, this test will tell you when it is unlikely that you have a normal distribution and is normally run using statistical software.
- Anderson-Darling tests are proposed for the continuous as well as discrete case.

Merits of Anderson-Darling (A-D test)

- It does not make any assumptions about the distribution of data.
- It is widely applicable not only in social sciences but in business research as well.
- There are no restrictions on the sample size. Small samples are acceptable.

A-D test for a goodness-of-fit test

- The Anderson-Darling statistic (A_2) is defined as

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}(2i-1)\cdot[\ln F(X_i) + \ln(1-F(X_{n-i+1}))]$$

- H_0: The data follow the specified distribution.
- H_A: The data do not follow the specified distribution.

A-D test for a goodness-of-fit test

- The hypothesis regarding the distributional form is rejected at the chosen significance level (alpha) if the test statistic A2 is greater than the critical value obtained from a table.

Example of Goodness of Fit Test

- Suppose you flip two coins 100 times.

  The results are 20 HH, 27 HT, 30 TH, and 23 TT.

  Are the coins fair?

  Test at a 5% significance level.

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Solution

- This problem can be set up as a goodness-of-fit problem. The sample space for flipping two fair coins is {HH, HT, TH, TT}.
- Out of 100 flips, you would expect 25 HH, 25 HT, 25 TH, and 25 TT.
- This is the expected distribution. The question, "Are the coins fair?" is the same as saying, "Does the distribution of the coins (20 HH, 27 HT, 30 TH, 23 TT) fit the expected distribution?"

Solution

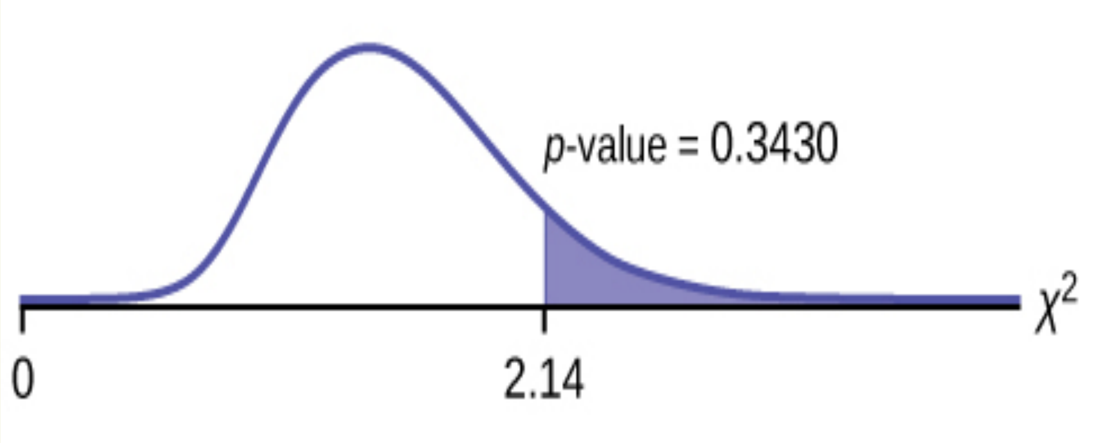
- Random Variable: Let X = the number of heads in one flip of the two coins.
- X takes on the values 0, 1, 2. (There are 0, 1, or 2 heads in the flip of two coins.) Therefore, the number of cells is three.
- Since X = the number of heads, the observed frequencies are 20 (for two heads), 57 (for one head), and 23 (for zero heads or both tails).

Solution

- The expected frequencies are 25 (for two heads), 50 (for one head), and 25 (for zero heads or both tails).
- This test is right-tailed.
- H_0: The coins are fair.
- H_a: The coins are not fair.

Solution

- Distribution for the test: χ^2_2 where df = 3 – 1 = 2.
- Calculate the test statistic: $\chi^2 = 2.14$.
- Graph:



Solution

- Probability statement:

$$\text{p-value} = P(\chi^2 > 2.14) = 0.3430$$

- Compare α and the p-value:

$\alpha = 0.05$
p-value = 0.3430
$\alpha <$ p-value.

Solution

- Make a decision:

  Since α < p-value, do not reject H_0.

- Conclusion:

  There is insufficient evidence to conclude that the coins are not fair.

References

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