ICCPP-STATISTICS
- Bartlett's Test

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M. S. Bartlett (1910-2002)

Bartlett's Test

Definition

- Bartlett's test is used to test if k samples have equal variances.
- Equal variances across samples is called homogeneity of variances.

Bartlett's Test

- Some statistical tests, for example the analysis of variance, assume that variances are equal across groups or samples.
- The Bartlett test can be used to verify that assumption.

Bartlett's Test

- Bartlett's test is sensitive to departures from normality.
- That is, if your samples come from non-normal distributions, then Bartlett's test may simply be testing for non-normality.

Hypothesis Testing

- Null hypothesis: Variance ($\sigma 2$) is equal across all groups.

  H_0: $\sigma^2_i = \sigma^2_j$ for all groups

- Alternative hypothesis: Variance is not equal across all groups.

  H_1: $\sigma^2_i \neq \sigma^2_j$ for at least one pair of groups

Hypothesis Testing

- Like many other techniques for testing hypotheses, Bartlett's test for homogeneity involves computing a test-statistic and finding the P-value for the test statistic, given degrees of freedom and significance level.
- If the P-value is smaller than the significance level, we reject the null hypothesis; if it is bigger, we cannot reject the null hypothesis.

How to Conduct Bartlett's Test

Step 1

- Specify the significance level (α).

How to Conduct Bartlett's Test

Step 2

- Compute the sample variance (s2j) for each group.

$$s^2_j = \frac{\sum_{j=1}^{k} (X_{i,j} - \bar{X}_j)^2}{(n_j - 1)}$$

How to Conduct Bartlett's Test

Step 2.1

- Where $X_{i,j}$ is the score for observation *i* in Group j, $\bar{X}_j$ is the mean of Group j, n_j is the number of observations in Group j , and k is the number of groups.

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How to Conduct Bartlett's Test

Step 3

- Compute the pooled estimate of sample variance (s^2p).

$$n = \sum n_i$$

$$s^2_p = \frac{\sum_{j=1}^{k} (n_j - 1) s^2_j}{(n - k)}$$

How to Conduct Bartlett's Test

Step 3.1

- Where n_j is the sample size in Group j , k is the number of groups, n is the total sample size, and s^2j is the sample variance in Group j.

Steps

Step 4

- Compute the test statistic (T).

$$A = (n - k) * \ln(s^2p)$$

$$B = \Sigma\ [(n_j - 1) * \ln(s^2j)]$$

$$C = 1 / [3 * (k - 1)]$$

$$D = \Sigma\ [1 / (n_j - 1) - 1 / (n - k)]$$

$$T = (A - B) / [1 + (C * D)]$$

Steps

Step 4

- Where A is the first term in the numerator of the test statistic, B is the second term in the numerator, C is the first term in the denominator, D is the second term in the denominator, and l_n is the natural logarithm.

Steps

Step 5

- Find the degrees of freedom (df), based on the number of groups (k).

$$df = k - 1$$

Steps

Step 6

- Find the P-value for the test statistic.
- The P-value is the probability of seeing a test statistic more extreme (bigger) than the observed T statistic from Step 4.

Steps

Step 6.1

- It turns out that the test statistic (T) is distributed much like a chi-square statistic with (k-1) degrees of freedom.
- Knowing the value of T and the degrees of freedom associated with T, we can use Stat Trek's Chi-Square Calculator to find the P-value - the probability of seeing a test statistic more extreme than T.

Steps

Step 7

- Accept or reject the null hypothesis, based on P-value and significance level.
- If the P-value is bigger than the significance level, we cannot reject the null hypothesis that variances are equal across groups.
- Otherwise, we reject the null hypothesis.

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References

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