ICCPP-STATISTICS
- A test

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Definition

- A-test is a statistical test which is used to check the mean difference of two dependent (correlating) samples for statistical significance.

Purpose

- The A-test is suitable for the two-sided test of the null hypothesis

$$H_o: \mu_1 - \mu_2 = \Delta = 0.$$

- The test statistic A is derived from the t-statistic, so that the A-test always leads to the same conclusions regarding the significance as the t-test for dependent samples.

A test

- Advantage

  The advantage of the A-test lies in the much simpler calculation of the test size.

Requirements

- There are two dependent samples;
  or to put it another way: there is a population of pairs of measured values
  and this population has become a random sample was drawn.

Requirements

- The differences between the pairs of measured values D_i (where $D_i = X_i1 - X_i2$) are normally distributed in the population.

Null hypothesis

- The two samples come from total populations with the same mean values.

$$H_o : \mu 1 = \mu 2 \; ; \text{or } \mu 1 - \mu 2 = \Delta = 0$$

Null hypothesis

- Or in other words:
  The sample with the mean D (D = mean of the differences = $x_{.1} - x_{.2}$) comes from a population with the mean $\Delta = 0$.

$$Ho : \Delta = 0$$

Alternative hypothesis

- $H_1 : \mu 1 \neq \mu 2$;

  or $\mu 1 - \mu 2 = \Delta \neq 0$

  or $\Delta \neq 0$

A test

- (A) Arrangement of the data

The data from the investigation can be arranged as follows:

Measured value pair no.	Condition (sample 1)	Condition (sample 2)	Difference between the pairs of measured values	
	Column 1	Column 2	Column 3	Column 4
1	$X_1 1$	$X_1 2$	$D_1 = X_1 1 - X_1 2$	
2	$X_2 1$	$X_2 2$	$D_2 = X_2 1 – X_2 2$	
•	•	•	•	•
•	•	•	•	•
•	•	•	•	•
1	$X_i 1$	$X_1 2$	$D_i = X_i 1 – X_i 2$	
•	•	•	•	•
•	•	•	•	•
•	•	•	•	•
n	$X_n 1$	$X_n 2$	$D_n = X_n 1 – X_n 2$	

A test

- (B) Overview of the calculations:

  The null hypothesis is tested against H1 using the following test statistic:

$$A = \frac{\sum_{i}^{n} D_i^2}{\left(\sum_{i}^{n} D_i\right)^2}$$

A test

- Where:

  $D_i = X_i1 - X_i2$ = difference between the measured values in the measured value pair (test subjects) i.

  Degree of freedom: $FG = n - 1$

  (where n = number of value pairs)

A test

- The null hypothesis is rejected if the calculated A-value is equal to or smaller than the table value in the A-table belonging to "a" and the degrees of freedom.

Calculation steps

For a clearer execution of the calculation steps 1-4 we create a table (like the one in section A).

Step 1

For each of the n pairs of measured values, form the difference Di from the two measured values, i. e.

Calculation steps

For each pair, subtract the measured value of term 2 (column 2 of the table) from the measured value under condition 2 (column 2 of the table) from the measured value under both conditions 2 (column 1 of the table).

The differences Di are entered in column 3 of the table.

$D_1 = X_1 1 - X_1 2, \ldots, D_i = X_i 1 - X_i 2, \ldots, D_n = X_n 1 - X_n 2$

+

Calculation steps

Step 2

Square the differences Di calculated at 1, i.e. square the values in column 3 of the table.

Enter the squared differences in column 4 of the table.

$$D_1^2, \ldots, D_i^2, \ldots, D_n^2$$

Calculation steps

Step 3

Add up the n differences calculated for 1, i.e. add up the values in column 3 of the table.

That is the sum of the differences.

$$\sum_{i}^{n} D_i$$

Calculation steps

Step 4

Add up the values D_i^2 calculated for 2, i.e. add up the values in column 4 of the table.

That gives the sum of the squared differences.

$$\sum_{i}^{n} D_i^2$$

Calculation steps

Step 5

Calculate the value of the test statistic A by plugging the results of 3 and 4 into the following formula 1:

$$A = \frac{(4)}{(3)^2} = \frac{\sum_{i}^{n} D_i^2}{\left(\sum_{i}^{n} D_i\right)^2}$$

Calculation steps

Step 6

Determine the number of degrees of freedom:

$$FG = n - 1$$

The A value calculated for 5 is now compared with the table value in the A table (Table 5). This table value is

$$A_a;\ n - 1$$

Calculation steps

Step 7

Decision about the null hypothesis

If the A value calculated at 5 is greater than the table attendant A_a ; n -1 ' then the null hypothesis is maintained.

Calculation steps

If the calculated A value is equal to or less than the table value A_a; n -1 ‘ then the null hypothesis is rejected.

This leads to the acceptance of the alternative hypothesis:

$$H_1 : \mu 1 - \mu 2 = \Delta \neq 0 .$$

References

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