ICCPP-STATISTICS
- Fisher Two Sample Randomization

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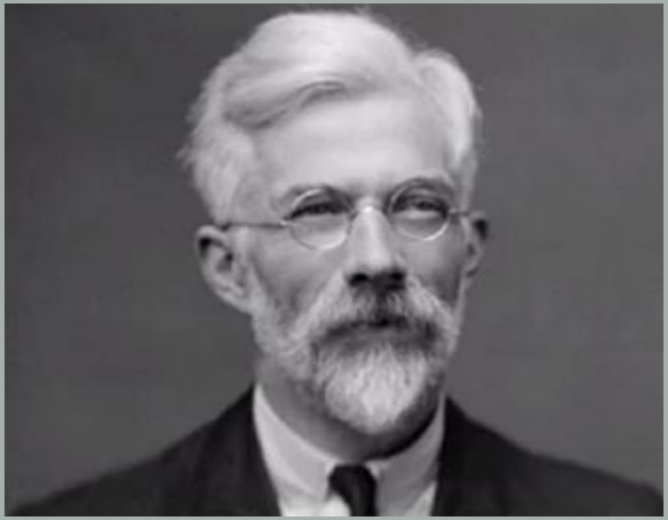

Ronald Fisher (1890-1962)

Fisher Two Sample Randomization

Purpose

- Perform a Fisher two sample randomization test for the equality of the means of two independent samples.

Description

The two sample t-test is the standard test for the equality of the means from two samples. This test is based on the following assumptions:

- The samples are randomly selected from infinite populations (equivalently the observations are independent)

Description

- The samples come from normal populations
- The two populations have equal variances

Randomization tests can be used when these assumptions are questionable.

Description

- Fisher introduced randomization tests (also referred to as permutation tests) in 1935.
- The randomization test for the equality of the means for two samples is computed as follows:

Description

Step 1

Given that sample one has n1 observations and sample two has n2 observations, randomly assign the n1 + n2 observations so that n1 observations are assigned to sample one and n2 observations and compute the difference of the means.

This is a single permutation for the test.

Description

Step 2

Generate all possible permutations of the $n1 + n2$ observations and compute the difference of the means for each permutation.

The number of permutations is

$$(n1\ n2) = n1!\ /\ n2!(n1 - n2)!.$$

Call this value NTOTAL for subsequent steps.

Description

Step 3

Let DFULL denote the difference of the means for the original samples.

Let D_i denote the difference of the means for the i-th sample.

Then the following p-values can be computed

Description

Step 3.1

$$p(\text{upper tail}) = \frac{\text{number of } D_i \geq \text{DFULL}}{\text{NTOTAL}}$$

$$p(\text{lower tail}) = \frac{\text{number of } D_i \leq \text{DFULL}}{\text{NTOTAL}}$$

$$p(\text{two tailed}) = \frac{\text{number of } |D_i| \geq \text{DFULL}}{\text{NTOTAL}}$$

Description

The primary drawback to this test is that NTOTAL grows rapidly as n1 and n2 increase.

A test based on the full set of permutations may be computationally prohibitive except for relatively small samples.

Description

For larger n1 and n2, one approach is to generate a random subset of the complete set of permutations (typically on the order of 4,000 to 10,000 random subsets will be generated).

For this command, Data plot is using the algorithm of Richards and Byrd. This algorithm generates the complete set of permutations.

Description

The advantage of this algorithm is that exact p-values are obtained for one-tailed tests and also for two-tailed tests when n1 = n2. If n1 is not equal n2, an approximate p-value is obtained for the two-tailed test.

The primary drawback is that this test is limited to small sample sizes.

Data plot currently limits the maximum value of n1 and n2 to be 22.

Description

If the two samples are not randomly drawn from larger populations, the inference will be valid for the observations under study but not necessarily for the populations from which the observations are drawn.

References

Richards and Byrd (1996), "Fisher's Randomization Test for Two Small Independent Samples", Applied Statistics, Vol. 45, No. 3, pp. 394-398.

Fisher (1935), "Design of Experiments", Edinburgh: Oliver and Boyd.

Conover (1999), "Practical Non-Parametric Statistics", Third Edition, Wiley, p. 410.

Higgins (2004), "Introduction to Modern Nonparametric Statistics", Thomson/Brooks/Cole, Duxbury Advanced Series, Chapter 2.

https://www.itl.nist.gov/div898/software/dataplot/refman1/auxillar/fishrand.htm