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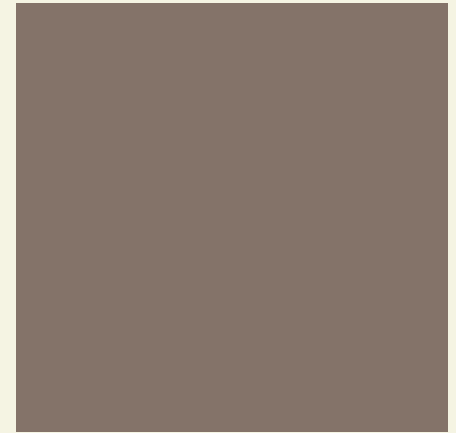
ICCPP-STATISTICS

- Two-way analysis of variance

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Ronald Fisher (1890-1962)

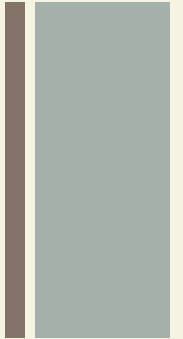
Two-way analysis of variance

+ Definition

- The two-way analysis of variance (ANOVA) is an extension of the one-way ANOVA that examines the influence of two different categorical independent variables on one continuous dependent variable.
- The two-way ANOVA not only aims at assessing the main effect of each independent variable but also if there is any interaction between them.

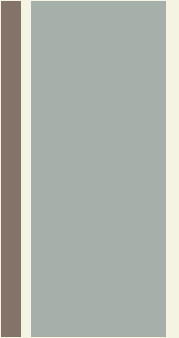
+ Uses

- Use a two way ANOVA when you have one measurement variable (i.e. a quantitative variable) and two nominal variables. In other words, if your experiment has a quantitative outcome and you have two categorical explanatory variables, a two way ANOVA is appropriate.
- You should have enough observations in your data set to be able to find the mean of the quantitative dependent variable at each combination of levels of the independent variables.



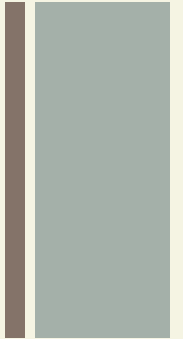
+ Uses

- A quantitative variable represents amounts or counts of things. It can be divided to find a group mean.
- You can use a two-way ANOVA when you have collected data on a quantitative dependent variable at multiple levels of two categorical independent variables.





Assumptions



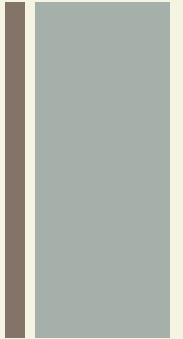
To use a two-way ANOVA your data should meet certain assumptions. Two-way ANOVA makes all of the normal assumptions of a parametric test of difference:

- Homogeneity of variance (a.k.a. homoscedasticity)

The variation around the mean for each group being compared should be similar among all groups.



Assumptions

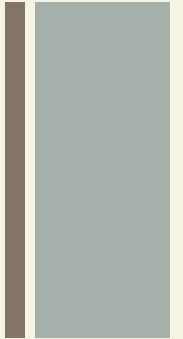


- Independence of observations

Your independent variables should not be dependent on one another (i.e. one should not cause the other). In addition, your dependent variable should represent unique observations – that is, your observations should not be grouped within locations or individuals.

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Assumptions



- Normally-distributed dependent variable

The values of the dependent variable should follow a bell curve.

+ Example

- Suppose you want to determine whether the brand of laundry detergent used and the temperature affects the amount of dirt removed from your laundry.
- To this end you buy two detergents with different brand (“Super” and “Best”) and choose three different temperature levels (“cold”, “warm” and “hot”).

+ Example

- Then you divide your laundry randomly into “6*r” pile of equal size and assign each ‘r’ piles into the combination of (“super” and “Best”) and (“cold”, “warm” and “hot”).

In this example, we are interested in testing Null Hypothesis.

- $H_0(D)$ = The amount of dirt removed does not depend on the type of detergent.
- $H_0(T)$ = The amount of dirt removed does not depend on the temperature.

+ Example

The example has two factors (factor detergent, factor temperature) at $a=2$ (Super and Best) and $b=3$ (cold, warm and hot) levels. Thus, there are $a*b = 3*2=6$ different combination of detergent and temperature with each combination.

There are $r=4$ loads. (r is called the number of replicates).

This sums up to “ $n=a*b*r$ ” = $24=2*3*4$ loads in total.

+ Example

- The amounts of $Y(ijk)$ of dirt removed when washing sub pile $k(k=1,2,3,4)$ with detergent $i(i=1,2)$ at temperature $j(j=1,2,3)$ are recorded in table below:

	cold	warm	hot
	4	7	10
	5	9	12
	6	8	11
Super	5	12	9
	6	13	12
	6	15	13
	4	12	10
Best	4	12	13

+ Solution Steps wise

	cold	warm	hot	M(d) [Y(i)]
	4	7	10	
	5	9	12	
	6	8	11	
Super	5	12	9	
	mean(Y _{ij})=5	mean(Y _{ij})=9	mean(Y _{ij})=10.5 ~10	8
	6	13	12	
	6	15	13	
	4	12	10	
Best	4	12	13	
	mean(Y _{ij})=5	mean(Y _{ij})=13	mean(Y _{ij})=12	10
M(t)[Y(j)]	5	11	11	9

+ Solution Steps wise

- We have calculated all the means like detergent mean(Md), temperature mean(Mt) and mean of every group combination.
- Now what we only have to do is calculate the sum of squares(ss) and degree of freedom(df) for temperature, detergent and interaction between factor and levels.
- First calculate the $SS(\text{within})/df(\text{within})$ we have already know how to calculate $SS(\text{within})/df(\text{within})$ in one way ANOVA we calculated this but in two way anova the formula is different.

+ Solution Steps wise

Step 1

Formula for calculation of SS(within) is:

- Y_{ijk} are the elements in the groups.
- $\bar{Y}(ij)$ is mean of combinations

+ Solution Steps wise

Step 1.1 When we put the values and do calculations with this formula we will get SS(within) is

$$SS_{within} = \sum_{i=1}^2 \sum_{j=1}^3 \sum_{k=1}^4 (Y_{ijk} - Y_{ij\cdot})^2$$

$$\begin{aligned} &= (4 - 5)^2 + (5 - 5)^2 + (6 - 5)^2 + (5 - 5)^2 + (7 - 9)^2 \\ &\quad + (9 - 9)^2 + (8 - 9)^2 + (12 - 9)^2 + (12 - 12)^2 \\ &\quad + (13 - 12)^2 + (10 - 12)^2 + (13 - 12) = \mathbf{38} \end{aligned}$$

+ Solution Steps wise

Step 1.2

- Calculate the $df(\text{within})$:

$$df(\text{within}) = (r-1)*a*b = 3*2*3 = \mathbf{18}$$

- Calculate $MS(\text{within})$:

$$MS(\text{within}) = SS(\text{within})/df(\text{within}) = 38/18 \\ = \mathbf{2.1111}$$

+ Solution Steps wise

Step 2

Calculate SS(detergent) and df(detergent) and MS(detergent)

- $\bar{Y}(i)$ is the mean of detergent
- \bar{Y} is the total mean detergent and temperature

+ Solution Steps wise

Step 2.1

$$SS_{detergent} = r \cdot b \cdot \sum_{i=1}^2 \left(\bar{Y}_{i..} - \bar{Y}_{...} \right)^2$$

$$= 4 \cdot 3 \left[(8-9)^2 + (10-9)^2 \right] = \mathbf{24}$$

■ Calculate df(detergent):

$$df(\text{detergent}) = a-1 = 2-1 = \mathbf{1}$$

+ Solution Steps wise

Step 2.2

- Calculate MS(detergent):

$$\begin{aligned} \text{MS(detergent)} &= \text{SS(detergent)} / \text{df(detergent)} \\ &= 24 / 1 = \mathbf{24} \end{aligned}$$

+ Solution Steps wise

Step 3

Calculate the $SS(\text{temperature})$, $df(\text{temperature})$ and $MS(\text{temperature})$

- $\bar{Y}(i)$ is the mean of detergent
- \bar{Y} is the total mean detergent and temperature

+ Solution Steps wise

Step 3.1

$$SS_{temperature} = r \cdot a \cdot \sum_{j=1}^3 (Y_{.j} - Y_{...})^2$$

$$= 4 \cdot 2 \cdot [(5 - 9)^2 + (11 - 9)^2 + (11 - 9)^2]$$

$$= 192$$

- Calculate $df(\text{temperature})$:

$$df(\text{temperature}) = b - 1 = 3 - 1 = 2$$

+ Solution Steps wise

Step 3.2

- Calculate MS(temperature):

$$\text{MS(temperature)} = \text{SS (temperature)} / \text{df (temperature)}$$

$$= 192/2 = \mathbf{81}$$

+ Solution Steps wise

Step 4

Calculate SS(interaction), df(interaction) and MS(interaction)

- $\bar{Y}(ij)$ is mean of combinations
- $\bar{Y}(i)$ is the mean of detergent
- $\bar{Y}(j)$ is the mean of temperature
- \bar{Y} is the total mean detergent and temperature

+ Solution Steps wise

Step 4.1

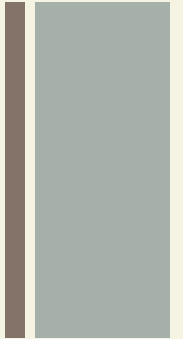
- Calculate SS(interaction):

$$SS_{interaction} = r \times \sum_{i=1}^2 \sum_{j=1}^3 (Y_{ij} - Y_{i.} - Y_{.j} + Y_{...})^2$$

$$= 4 \times (5 - 8 - 5 + 9)^2 + (9 - 8 - 11 + 9)^2 + (110 - 8 - 11 + 9)^2 + \dots + (12 - 11 - 10 + 9)^2$$

$$= 12$$

+ Solution Steps wise



Step 4.2

- Calculate $df(\text{interaction})$:

$$\begin{aligned}df(\text{interaction}) &= (a-1)*(b-1) \\ &= (2-1)*(3-1) = \mathbf{2}\end{aligned}$$

- Calculate $MS(\text{interaction})$:

$$\begin{aligned}MS(\text{interaction}) &= SS(\text{interaction})/df(\text{interaction}) \\ &= 12/2 = \mathbf{6}\end{aligned}$$

+ Solution Steps wise

- Its time to calculate the F-test:
Calculate critical F-value

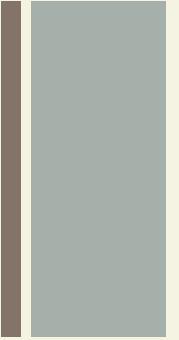
$MS(\text{detergent})/MS(\text{within}) \sim$
 $F(df(\text{detergent}),$

$df(\text{within})) MS(\text{temperature})/MS(\text{within}) \sim$
 $F(df(\text{temperature}),$

$df(\text{within})) MS(\text{interaction})/MS(\text{within}) \sim$
 $F(df(\text{interaction}), df(\text{within}))$

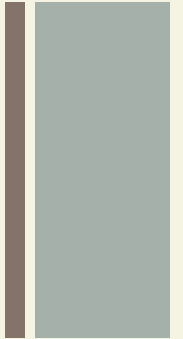
+ Solution Steps wise

- If you found the F-value less than the critical F-value then you will not be able to reject the null hypothesis.





References



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