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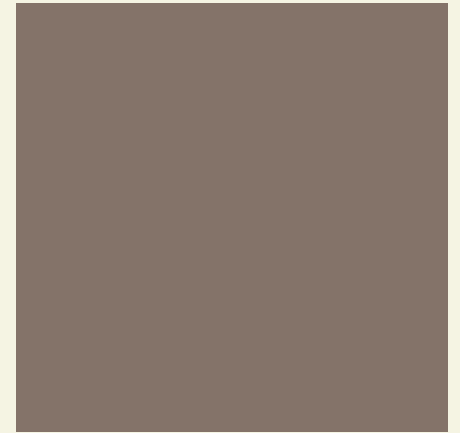
ICCPP-STATISTICS

- One-way analysis of variance

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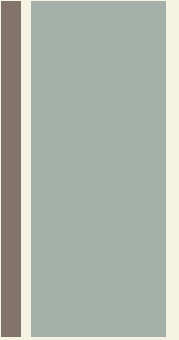


Ronald Fisher (1890-1962)

One-way analysis of variance

+ Definition

- One-way analysis of variance (abbreviated one-way ANOVA) is a technique that can be used to compare whether two samples means are significantly different or not (using the F distribution).

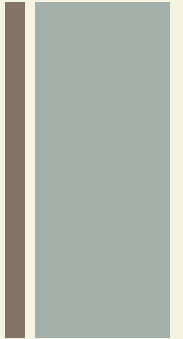


+ Uses

- A one way ANOVA is used to compare two means from two independent (unrelated) groups using the F-distribution.

The null hypothesis for the test is that the two means are equal. Therefore, a significant result means that the two means are unequal.

+ Uses



- The one-way ANOVA compares the means between the groups you are interested in and determines whether any of those means are statistically significantly different from each other.

+ Uses

- Specifically, it tests the null hypothesis:

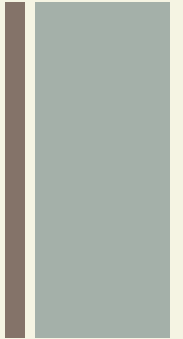
$$H_0: \mu_1 = \mu_2 = \mu_3 = \dots = \mu_k$$

- where μ = group mean and k = number of groups.

If, however, the one-way ANOVA returns a statistically significant result, we accept the alternative hypothesis (H_A), which is that there are at least two group means that are statistically significantly different from each other.



Assumptions

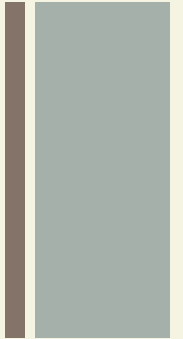


The results of a one-way ANOVA can be considered reliable as long as the following assumptions are met:

- Response variable residuals are normally distributed (or approximately normally distributed).
- Variances of populations are equal.



Assumptions



- Responses for a given group are independent and identically distributed normal random variables (not a simple random sample (SRS)).
- If data are ordinal, a non-parametric alternative to this test should be used such as Kruskal–Wallis one-way analysis of variance. If the variances are not known to be equal, a generalization of 2-sample Welch's t-test can be used.

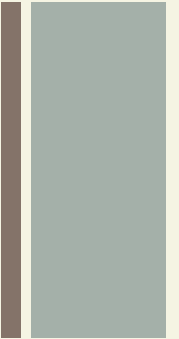
+ Example

- Suppose we want to know whether or not three different exam prep programs lead to different mean scores on a certain exam.

To test this, we recruit 30 students to participate in a study and split them into three groups.

The students in each group are randomly assigned to use one of the three exam prep programs for the next three weeks to prepare for an exam.

At the end of the three weeks, all of the students take the same exam.



+ Example

- The exam scores for each group are shown below:

Group 1	Group 2	Group 3
85	91	79
86	92	78
88	93	88
75	85	94
78	87	92
94	84	85
98	82	83
79	88	85
71	95	82
80	96	81

+ Solution Steps wise

Step 1

Calculate the group means and the overall mean.

First, we will calculate the mean for all three groups along with the overall mean:

	Group 1	Group 2	Group 3
	85	91	79
	86	92	78
	88	93	88
	75	85	94
	78	87	92
	94	84	85
	98	82	83
	79	88	85
	71	95	82
	80	96	81
Group Means	83.4	89.3	84.7
Overall Mean	85.8		

+ Solution Steps wise

Step 2

Calculate SSR.

Next, we will calculate the regression sum of squares (SSR) using the following formula:

$$n \sum (X_j - \bar{X}_{..})^2$$

- **n**: the sample size of group j
- **Σ** : a greek symbol that means “sum”
- **X_j** : the mean of group j
- **$\bar{X}_{..}$** : the overall mean

+ Solution Steps wise

Step 2.1

In our example, we calculate that

$$\text{SSR} = 10(83.4-85.8)^2 + 10(89.3-85.8)^2 + 10(84.7-85.8)^2 = 192.2$$

+ Solution Steps wise

Step 3

Calculate SSE

Next, we will calculate the error sum of squares (SSE) using the following formula:

$$\Sigma(X_{ij} - \bar{X}_j)^2$$

where:

- **Σ** : a greek symbol that means “sum”
- **X_{ij}** : the i^{th} observation in group j
- **\bar{X}_j** : the mean of group j

+ Solution Steps wise

Step 3.1

In our example, we calculate SSE as follows:

$$\text{Group 1: } (85-83.4)^2 + (86-83.4)^2 + (88-83.4)^2 + (75-83.4)^2 + (78-83.4)^2 + (94-83.4)^2 + (98-83.4)^2 + (79-83.4)^2 + (71-83.4)^2 + (80-83.4)^2 = 640.4$$

$$\text{Group 2: } (91-89.3)^2 + (92-89.3)^2 + (93-89.3)^2 + (85-89.3)^2 + (87-89.3)^2 + (84-89.3)^2 + (82-89.3)^2 + (88-89.3)^2 + (95-89.3)^2 + (96-89.3)^2 = 208.1$$

$$\text{Group 3: } (79-84.7)^2 + (78-84.7)^2 + (88-84.7)^2 + (94-84.7)^2 + (92-84.7)^2 + (85-84.7)^2 + (83-84.7)^2 + (85-84.7)^2 + (82-84.7)^2 + (81-84.7)^2 = 252.1$$

$$\text{SSE: } 640.4 + 208.1 + 252.1 = 1100.6$$

+ Solution Steps wise

Step 4

Calculate SST

Next, we will calculate the total sum of squares (SST) using the following formula:

$$SST = SSR + SSE$$

In our example, $SST = 192.2 + 1100.6 = \mathbf{1292.8}$

+ Solution Steps wise

Step 5

Fill in the ANOVA table.

Now we have SSR, SSE, and SST, we can fill in the ANOVA table:

Source	Sum of Squares (SS)	df	Mean Squares (MS)	F
Treatment	192.2	2	96.1	2.358
Error	1100.6	27	40.8	
Total	1292.8	29		

+ Solution Steps wise

Step 5.1

Here is how we calculated the various numbers in the table

- df treatment $k-1 = 3-1 = 2$
- df error $n-k = 30-3 = 27$
- df total $n-1 = 30-1 = 29$
- MS treatment $SST / df \text{ treatment} = 192.2 / 2 = 96.1$
- MS error $SSE / df \text{ error} = 1100.6 / 27 = 40.8$
- F $MS \text{ treatment} / MS \text{ error} = 96.1 / 40.8 = 2.358$

Note: n = total observations, k = number of groups

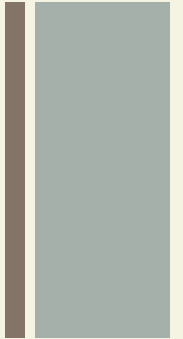
+ Solution Steps wise

Step 6

Interpret the results

The F test statistic for this one-way ANOVA is 2.358.

To determine if this is a statistically significant result, we must compare this to the F critical value found in the F distribution table with the following values:



+ Solution Steps wise

Step 6.1

- α (significance level) = 0.05
- DF1 (numerator degrees of freedom) = df treatment = 2
- DF2 (denominator degrees of freedom) = df error = 27

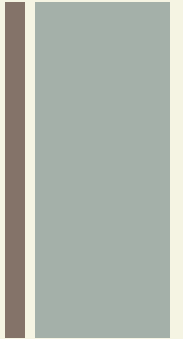
We find that the F critical value is **3.3541**.

+ Solution Steps wise

- Since the F test statistic in the ANOVA table is less than the F critical value in the F distribution table, we fail to reject the null hypothesis.
- This means we don't have sufficient evidence to say that there is a statistically significant difference between the mean exam scores of the three groups.



References



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