# ICCPP-STATISTICS

## - Dependent Samples T Test


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# William Sealy Gosset (1876-1937)

## Dependent Samples T Test

# Definition

- A paired t test (also called a correlated pairs t-test, a paired samples t test or dependent samples t test) is where you run a t test on dependent samples.

  Dependent samples are essentially connected — they are tests on same persons or things.

# Dependent Samples T Test

$$t = \frac{(\sum D)/N}{\sqrt{\frac{\sum D^2 - \left(\frac{(\sum D)^2}{N}\right)}{(N-1)(N)}}}$$

| | |
|---|---|
| $\sum D$: | Sum of the differences |
| $\sum D2$: | Sum of the squared differences |
| $(\sum D)2$: | Sum of the differences, squared |

# T Test types

- One sample t test: used to compare a result to an expected value.
- Paired t test (dependent samples): used to compare related observations.

# Dependent Samples T Test

- A paired-samples (correlated-samples or dependent-samples) is used when you have one sample of subjects who are tested several times, but under different conditions, that is, under different levels of an independent variable.

  Each subject is measured on the same dependent variable, but under different levels of an independent variable and you compare performance of the subjects between the different levels of this independent variable (with-subjects design).

# The paired samples t-test is used when

- The dependent variable is quantitative in nature
- The independent variable is qualitative in nature, that is, the levels represent different categories.
- The independent variable has two and only two levels.
- The independent variable is manipulated within-subjects

# The paired samples t-test is used when

- Choose the paired t-test if you have two measurements on the same item, person or thing. You should also choose this test if you have two items that are being measured with a unique condition.
- For example: Knee MRI costs at two different hospitals.

# Example

Calculate a paired t test by hand for the following data

| Subject # | Score 1 | Score 2 |
|---|---|---|
| 1 | 3 | 20 |
| 2 | 3 | 13 |
| 3 | 3 | 13 |
| 4 | 12 | 20 |
| 5 | 15 | 29 |
| 6 | 16 | 32 |
| 7 | 17 | 23 |
| 8 | 19 | 20 |
| 9 | 23 | 25 |
| 10 | 24 | 15 |
| 11 | 32 | 30 |

# Solution Step Wise

## Step 1

Subtract each Y score from each X score.

| Subject # | Score 1 | Score 2 | X-Y |
|---|---|---|---|
| 1 | 3 | 20 | -17 |
| 2 | 3 | 13 | -10 |
| 3 | 3 | 13 | -10 |
| 4 | 12 | 20 | -8 |
| 5 | 15 | 29 | -14 |
| 6 | 16 | 32 | -16 |
| 7 | 17 | 23 | -6 |
| 8 | 19 | 20 | -1 |
| 9 | 23 | 25 | -2 |
| 10 | 24 | 15 | 9 |
| 11 | 32 | 30 | 2 |

# Solution Step Wise

Step 2 Add up all of the values from Step 1.

Set this number aside for a moment.

| Subject # | Score 1 | Score 2 | X-Y |
|---|---|---|---|
| 1 | 3 | 20 | -17 |
| 2 | 3 | 13 | -10 |
| 3 | 3 | 13 | -10 |
| 4 | 12 | 20 | -8 |
| 5 | 15 | 29 | -14 |
| 6 | 16 | 32 | -16 |
| 7 | 17 | 23 | -6 |
| 8 | 19 | 20 | -1 |
| 9 | 23 | 25 | -2 |
| 10 | 24 | 15 | 9 |
| 11 | 32 | 30 | 2 |
| | | **SUM:** | -73 |

# Solution Step Wise

## Step 3

Square the differences from Step 1.

| Subject # | Score 1 | Score 2 | X-Y | (X-Y)^2 |
|---|---|---|---|---|
| 1 | 3 | 20 | -17 | 289 |
| 2 | 3 | 13 | -10 | 100 |
| 3 | 3 | 13 | -10 | 100 |
| 4 | 12 | 20 | -8 | 64 |
| 5 | 15 | 29 | -14 | 196 |
| 6 | 16 | 32 | -16 | 256 |
| 7 | 17 | 23 | -6 | 36 |
| 8 | 19 | 20 | -1 | 1 |
| 9 | 23 | 25 | -2 | 4 |
| 10 | 24 | 15 | 9 | 81 |
| 11 | 32 | 30 | 2 | 4 |
| | | SUM: | -73 | |

# Solution Step Wise

## Step 4

Add up all of the squared differences from Step 3.

| Subject # | Score 1 | Score 2 | X-Y | (X-Y)^2 |
|---|---|---|---|---|
| 1 | 3 | 20 | -17 | 289 |
| 2 | 3 | 13 | -10 | 100 |
| 3 | 3 | 13 | -10 | 100 |
| 4 | 12 | 20 | -8 | 64 |
| 5 | 15 | 29 | -14 | 196 |
| 6 | 16 | 32 | -16 | 256 |
| 7 | 17 | 23 | -6 | 36 |
| 8 | 19 | 20 | -1 | 1 |
| 9 | 23 | 25 | -2 | 4 |
| 10 | 24 | 15 | 9 | 81 |
| 11 | 32 | 30 | 2 | 4 |
| | | **SUM:** | -73 | **1131** |

# Solution Step Wise

## Step 5

Use the following formula to calculate the t-score

$$t = \frac{(\sum D)/N}{\sqrt{\frac{\sum D^2 - \left(\frac{(\sum D)^2}{N}\right)}{(N-1)(N)}}}$$

$\sum D$: Sum of the differences (Sum of X-Y from Step 2)

$\sum D2$: Sum of the squared differences (from Step 4)

$(\sum D)2$: Sum of the differences (from Step 2), squared

# Solution Step Wise

$$t = \frac{(\sum D)/N}{\sqrt{\frac{\sum D^2 - \left(\frac{(\sum D)^2}{N}\right)}{(N-1)(N)}}}$$

$$t = \frac{-73/11}{\sqrt{\frac{1131 - \left(\frac{(-73)^2}{11}\right)}{(11-1)(11)}}}$$

$$t = \frac{-73/11}{\sqrt{\frac{1131 - \left(\frac{5329}{11}\right)}{110}}}$$

$$t = -\ 2.74$$

# Solution Step Wise

Step 6

Subtract 1 from the sample size to get the degrees of freedom. We have 11 items, so 11-1 = 10.

# Solution Step Wise

Step 7

Find the p-value in the t-table, using the degrees of freedom in Step 6.

If you don't have a specified alpha level, use 0.05 (5%).

For this example problem, with df = 10,
the t-value is 2.228.

# Solution Step Wise

Step 8

Compare your t-table value from Step 7 (2.228) to your calculated t-value (-2.74).

The calculated t-value is greater than the table value at an alpha level of .05.

The p-value is less than the alpha level: $p < .05$.

We can reject the null hypothesis that there is no difference between means.

# Solution Step Wise

**Note**

You can ignore the minus sign when comparing the two t-values, as ± indicates the direction; the p-value remains the same for both directions.

# References

Goulden, C. H. Methods of Statistical Analysis, 2nd ed. New York: Wiley, pp. 50-55, 1956.

Lüroth, J. (1876). "Vergleichung von zwei Werthen des wahrscheinlichen Fehlers". Astronomische Nachrichten (in German). 87 (14): 209–220. Bibcode:1876AN.....87..209L. doi:10.1002/asna.18760871402

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