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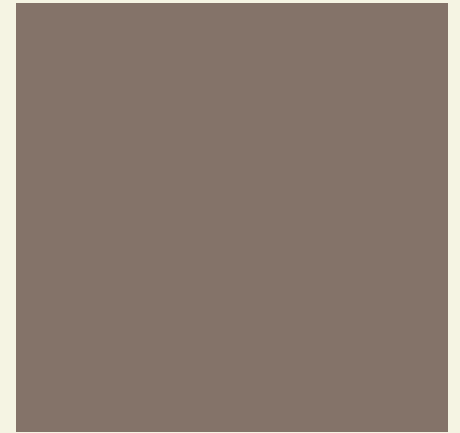
# ICCPP-STATISTICS

- Wilcoxon signed rank test

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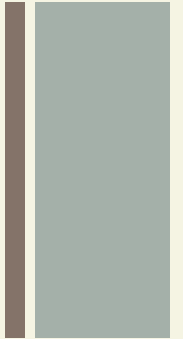


## Frank Wilcoxon (1892-1965)

Wilcoxon signed rank test



# Definition



- The Wilcoxon test, which can refer to either the Rank Sum test or the Signed Rank test version, is a nonparametric statistical test that compares two paired groups.

# + Wilcoxon signed rank test

$$W = \sum_{i=1}^{N_r} [\text{sgn}(x_{2,i} - x_{1,i}) \cdot R_i]$$

$W$  = test statistic

$N_r$  = sample size, excluding pairs where  $x_1 = x_2$

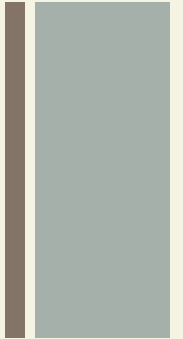
$\text{sgn}$  = sign function

$x_{1,i}, x_{2,i}$  = corresponding ranked pairs from two distributions

$R_i$  = rank  $i$



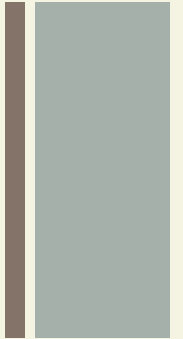
# Test Statistics



- The test statistic for the Wilcoxon Signed Rank Test is  $W$ , defined as the smaller of  $W^+$  (sum of the positive ranks) and  $W^-$  (sum of the negative ranks).

If the null hypothesis is true, we expect to see similar numbers of lower and higher ranks that are both positive and negative (i.e.,  $W^+$  and  $W^-$  would be similar).

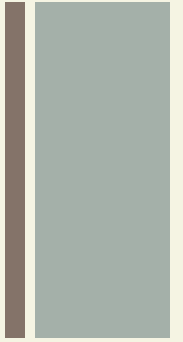
# + Assumptions



1.

Data are paired and come from the same population.

# + Assumptions



2.

Each pair is chosen randomly and independently.

# + Assumptions

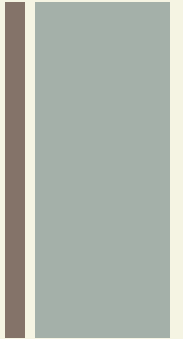
## 3.

The data are measured on at least an interval scale when, as is usual, within-pair differences are calculated to perform the test (though it does suffice that within-pair comparisons are on an ordinal scale)..





# Requirements for the test



- Data must be matched.
- The dependent variable must be continuous (i.e. you must be able to distinguish between values at the  $n$ th decimal place).
- You should have no tied ranks for maximum accuracy. If ranks are tied, there is a workaround.

# + Example

Is there a difference between the median values for the following sets of treatment data for the twelve groups?

Group #	Treatment 1	Treatment 2
1	2.5	4.0
2	3.5	5.6
3	2.9	3.2
4	2.1	1.9
5	6.9	9.5
6	2.4	2.3
7	4.9	6.7
8	6.6	6.0
9	2.0	3.5
10	2.0	4.0
11	5.8	8.1
12	7.5	19.9

# + Solution Step Wise

## Step 1

Subtract treatment 2 from treatment 1 to get the differences:

Group #	Treatment 1	Treatment 2	Difference
1	2.5	4.0	1.5
2	3.5	5.6	2.1
3	2.9	3.2	0.3
4	2.1	1.9	-0.2
5	6.9	9.5	2.6
6	2.4	2.3	-0.1
7	4.9	6.7	1.8
8	6.6	6.0	-0.6
9	2.0	3.5	1.5
10	2.0	4.0	2.0
11	5.8	8.1	2.3
12	7.5	19.9	12.4

# + Solution Step Wise

## Step 2

Place the differences in order (column 2 in the picture below), and then rank them. Ignore the sign when placing in rank order.

Rank	<u>Ds</u> in Order
1	0.1
2	0.2
3	0.3
4	0.6
5	1.5
6	1.5
7	1.8
8	2.0
9	2.1
10	2.3
11	2.6
12	12.4

# + Solution Step Wise

## Step 3

Make a third column and note the sign of the difference.

Rank	<u>Ds</u> in Order	Sign
1	0.1	-
2	0.2	-
3	0.3	+
4	0.6	-
5	1.5	+
6	1.5	+
7	1.8	+
8	2.0	+
9	2.1	+
10	2.3	+
11	2.6	+
12	12.4	+

# + Solution Step Wise

**The next two steps calculate the Wilcoxon signed rank sums**

## **Step 4**

Calculate the sum of the ranks of the negative differences (the ones with the negative sign in the Step 3 chart). You're adding up the ranks here, not the actual differences:

$$W_- = 1 + 2 + 4 = 7$$

# + Solution Step Wise

## Step 5

Calculate the sum of the ranks of the positive differences (the ones with the positive sign in the Step 3 chart).

$$W+ = 3 + 5.5 + 5.5 + 7 + 8 + 9 + 10 + 11 + 12 = 71$$



# References

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