ICCPP-STATISTICS

- Normal Distribution

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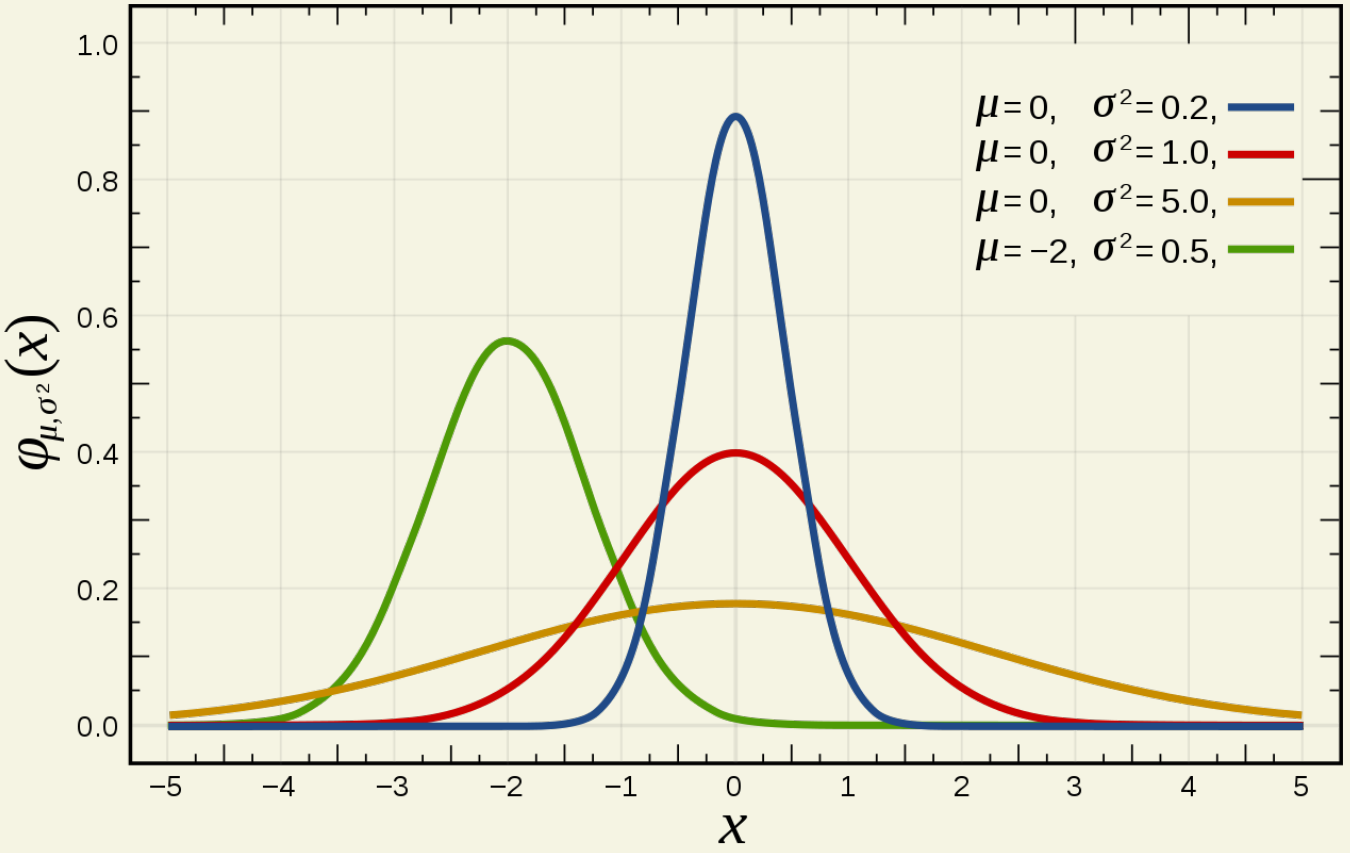
Carl Friedrich Gauß (1777 - 1855)

Normal distributions

Four Normal Distributions



Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

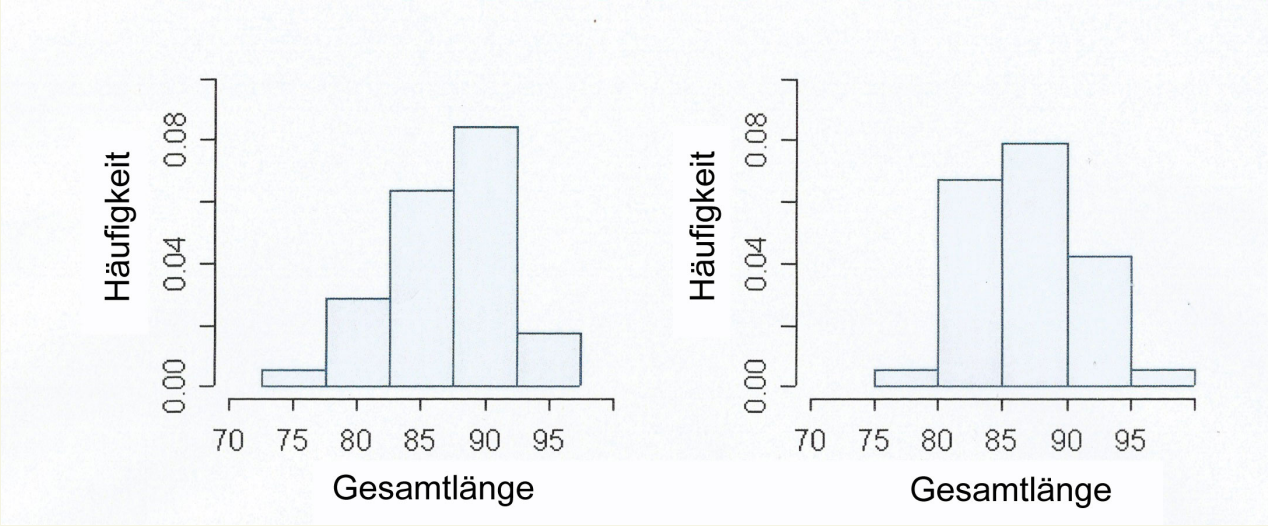
Importance of normal distribution

- The normal distribution is a significant tool in the hands of teacher and researcher of psychology and education.

  Through which he can decide the nature of the distribution of the scores obtained on the basis of measured variable.

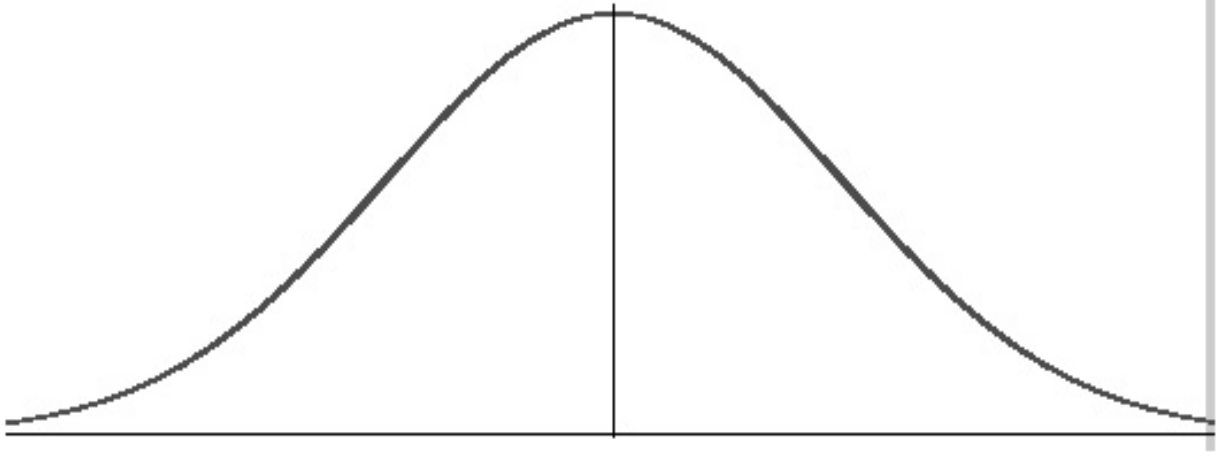
Histograms are not enough to check the normal distribution

The appearance of a histogram depends on the bar or class width and the class boundaries.



Definition

A Function that represents the distribution of many random variables as a symmetrical bell-shaped graph.



A normal distribution.

Properties of a normal distribution

- The mean, mode and median are all equal.
- The curve is symmetric at the center (i.e. around the mean, μ).
- Exactly half of the values are to the left of center and exactly half the values are to the right.
- The total area under the curve is 1.

Normal Distribution Formula

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$f(x)$ = probability density function

σ = standard deviation

μ = mean

Most used distribution

- The Normal distribution is by far the most used distribution in inferential statistics because of the following reasons:

1.

Number of evidences are accumulated to show that normal distribution provides a good fit or describe the frequencies of occurrence of many variable facts in biological statistics, e.g. sex ratio in births, in a country over a number of years. The anthropometrical data, e.g. height, weight, etc. The social and economic data e.g. rate of births, marriages and deaths. In psychological measurements e.g. Intelligence, perception span, reaction time, adjustment, anxiety etc. In errors of observation in physics, chemistry, astronomy and other physical sciences.

Most used distribution

2.

The normal distribution is of great value in educational evaluation and educational research, when we make use of mental measurement. It may be noted that normal distribution is not an actual distribution of scores on any test of ability or academic achievement, but is, instead, a mathematical model. The distributions of test scores approach the theoretical normal distribution as a limit, but the fit is rarely ideal and perfect.

Most used distribution

- To determine the percentage of cases in a normal distribution within given limits of scores.

Often a psychometrician or psychology teacher is interested to know the number of cases or individuals that lie in between two points or two limits. For example, a teacher may be interested as to how many students of his class got marks in between 60% and 70% in the annual examination, or he may be interested in how many students of his got marks above 80%.

Most used distribution

- **Drawing a normal distribution example**

  - The trunk diameter of a certain variety of pine tree is normally distributed with a mean of μ = 150cm and a standard deviation of σ = 30cm. **Sketch a normal curve that describes this distribution.**

Most used distribution

- Solution

Step 1

Sketch a normal curve.

Step 2

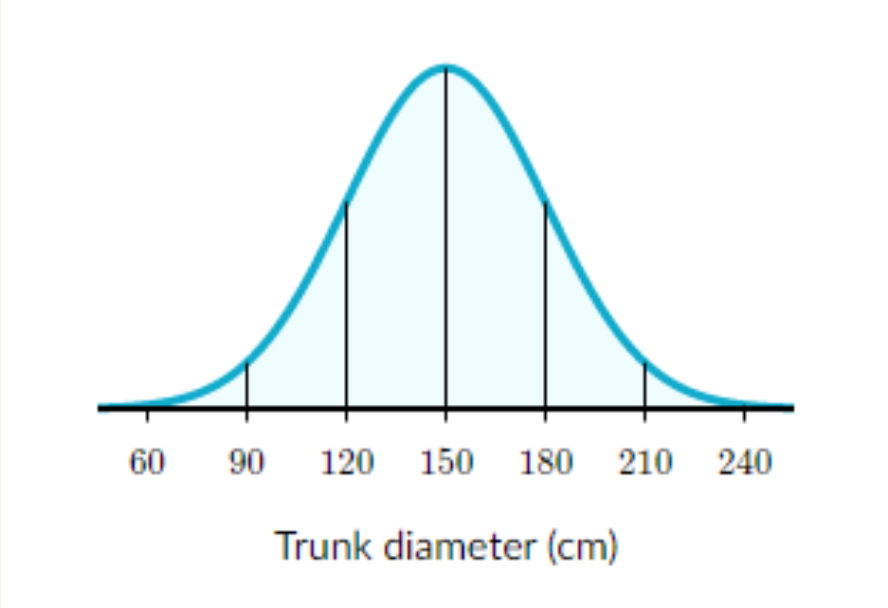
The mean of 150cm goes into the middle.

Most used distribution

- Solution

Step 3

Each standard deviation is a distance of 30cm.



Most used distribution

- Finding percentages example

A certain variety of pine tree has a mean trunk diameter of μ = 150cm and a standard deviation of σ = 30cm.

Approximately what percent of these trees have a diameter greater than 210cm?

Most used distribution

- Solution

Step 1

Sketch a normal distribution with a mean of $\mu = 150$cm and a standard deviation of $\sigma = 30$cm.

Step 2

The diameter of 210cm is two standard deviations above the mean. Shade above that point.

Most used distribution

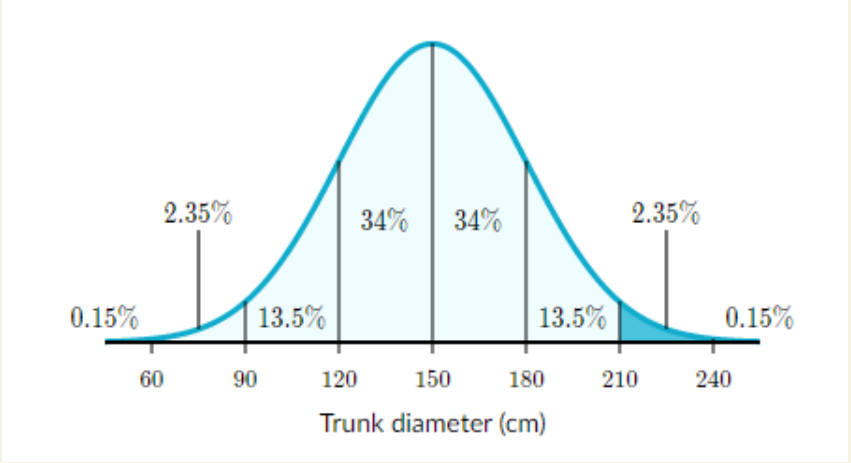
- Solution

Step 3

Add the percentages in the shaded area

2.35% + 0.15% =

2.15%



About 2.5% of these trees have a diameter greater than 210cm.

Most used distribution

- **Finding a whole count example**

  - A certain variety of pine tree has a mean trunk diameter of μ = 150cm and a standard deviation of σ = 30cm. A certain section of a forest has 500 of these trees. **Approximately how many of these trees have a diameter smaller than 120cm?**

Most used distribution

- Solution

Step 1

Sketch a normal distribution with a mean of $\mu = 150$cm and a standard deviation of $\sigma = 30$cm.

Step 2

The diameter of 210cm is two standard deviations above the mean. Shade above that point.

Most used distribution

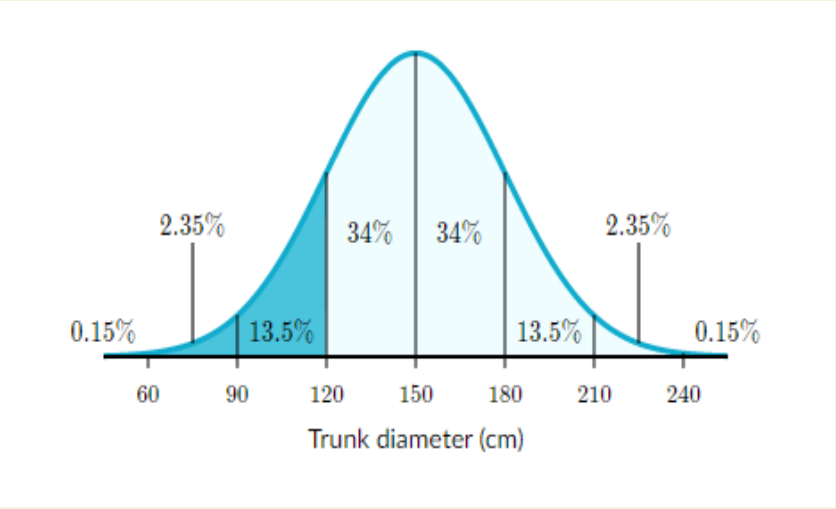
- Solution

Step 3

Add the percentages in the shaded area

0.15% + 2.35% + 13.5% =

16%



About 16% of these trees have a diameter smaller than 120cm.

Most used distribution

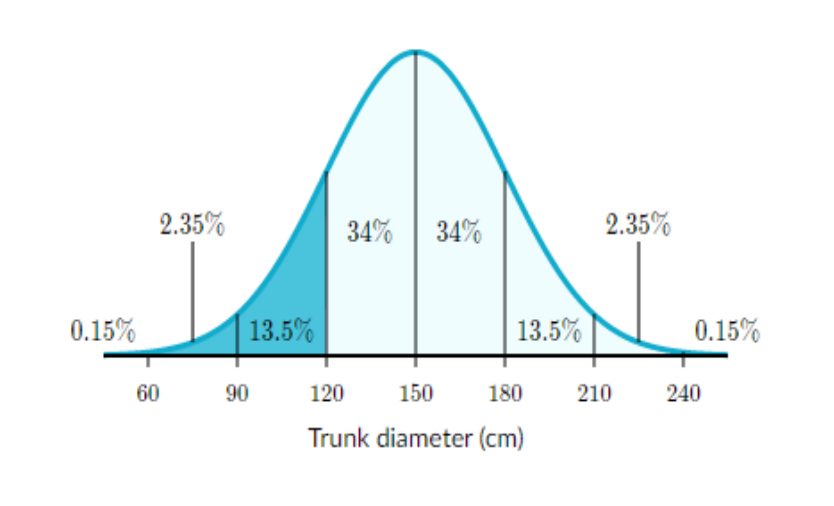
- Solution

Step 4

Find how many trees in the forest that percent represents.

16% of 500 = 0.16 * 500 =

80



About 80 of these trees have a diameter smaller than 120cm.

References

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[4] Stephanie Glen. "Welcome to Statistics How To!" From StatisticsHowTo.com: Elementary Statistics for the rest of us! https://www.statisticshowto.com/